Recursive Timed Automata (for CS 713 -Special Topics in Automata and Logics)

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Recursive Timed Automata

CS 713 - Special Topics in Automata and Logics

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Outline

1. Recursive State Machines
   - Labelled Transition Systems
   - Games on Labelled Transition Systems
   - Recursive State Machines
   - Semantics of RSM
2. Recursive Timed Automata
   - Definition
   - RTA semantics
3. Undecidability of RTA Termination with ≥ 3 clocks
   - Two counter Minsky machines
   - Gadgets DB, HF, P3O, P2O, $\text{HALT}^{\mathcal{A}}$
4. Decidability of Reachability on Glitch-free RTA
   - Glitch-free RTA
   - Region Equivalence

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Labelled Transition Systems

A labelled transition system (LTS) is a tuple $\mathcal{L} = (S, A, X)$, where

- S is the set of states,
- A is the set of actions, and
- $X : S \times A \to S$ is the transition function.

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Games on Labelled Transition Systems

A game arena G is a tuple $(\mathcal{L}, S_{\text{Ach}}, S_{\text{Tor}})$, where

- $\mathcal{L} = (S, A, X)$ is an LTS,
- $S_{\mathrm{Ach}} \subseteq S$ is the set of states controlled by player Achilles, and
- $S_{\text{Tor}} \subseteq S$ is the set of states controlled by player Tortoise

A strategy of player Achilles is a partial function $\alpha : FRuns^{\mathcal{L}} \to A$ such that for a run $r \in FRuns^{\mathcal{L}}$ we have that $\alpha(r)$ is defined if $\text{last}(r) \in S_{\text{Ach}}$, and $\alpha(r) \in A(\text{last}(r))$ for every such r. A strategy of player Tortoise is defined analogously. For an initial state s and a set of final states F, the value $\text{Val}_F^{\mathcal{L}}(s)$ of the reachability game is defined the number of transitions that Tortoise can ensure before the game visits a state in F irrespective of the strategy of Achilles.
Achilles wins the reachability game if $\text{Val}_F^{\mathcal{C}}(s) < \infty$

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Recursive State Machines

A recursive state machine $\mathcal{M} = (\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_k)$ is a tuple of components, where for each $1 \leq i \leq k$ component $\mathcal{M}_i = (N_i, En_i, Ex_i, B_i, Y_i, A_i, X_i)$ consists of:

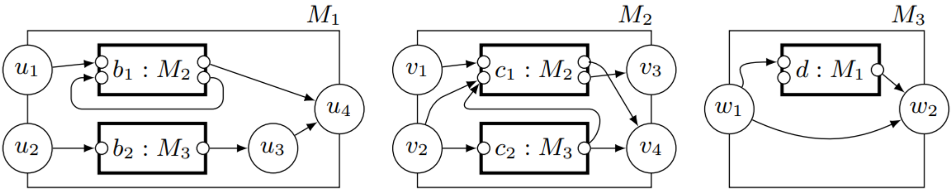
- a finite set N_i of nodes, including the set En_i entry nodes and the set Ex_i of exit nodes.
- a finite set B_i of boxes.
- boxes-to-components mapping $Y_i : B_i \to \{1, 2, \ldots, k\}$ that assigns every box to *a* component. To each box $b \in B_i$ we associate a set of call ports Call(b), and a set of return ports Ret(b) :

$$\text{Call}(b) = \left\{(b, en) : \text{en} \in En_{Y_i(b)}\right\}$$

$$\text{Ret}(b) = \left\{(b, ex) : \text{ex} \in Ex_{Y_i(b)}\right\}$$

  $Q_i = N_i \cup Call_i \cup \text{Ret}_i$
- a finite set A_i of actions.
- a transition function $X_i : Q_i \times A_i \to Q_i$ with a condition that call ports and exit nodes do not have any outgoing transitions.

Example



$$\mathcal{M}_i = (N_i, En_i, Ex_i, B_i, Y_i, A_i, X_i)$$

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Semantics of RSM

Let $\mathcal{M} = (\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_k)$ be an RSM where the component $\mathcal{M}_i$ is $(N_i, En_i, Ex_i, B_i, Y_i, A_i, X_i)$. The semantics of $\mathcal{M}$ is the countable labelled transition system$[\mathcal{M}] = (S_\mathcal{M}, A_\mathcal{M}, X_\mathcal{M})$ where:

- $S_\mathcal{M} \subseteq B^* \times Q$ is the set of states;
- $A_\mathcal{M} = \cup_{i=1}^{k} A_i$ is the set of actions;
- $X_\mathcal{M} : S_\mathcal{M} \times A_\mathcal{M} \to S_\mathcal{M}$ is the transition function such that for $s = (\langle \kappa \rangle, q) \in S_\mathcal{M}$ and $a \in A_\mathcal{M}$, we have that $s' = X_\mathcal{M}(s, a)$ if and only if one of the following holds:
  - the vertex q is a call port, i.e. $q = (b, en) \in Call$, and $s' = (\langle \kappa, b \rangle, en)$;
  - the vertex q is an exit node, i.e. $q = ex \in Ex$ and $s' = (\langle \kappa' \rangle, (b, ex))$ where $(b, ex) \in \mathrm{Ret}(b)$ and $\kappa = (\kappa', b)$;
  - the vertex q is any other kind of vertex, and $s' = (\langle \kappa \rangle, q')$ and $q' \in X(q, a)$.

Games on RSM: $([\mathcal{M}], [Q_{\mathrm{Ach}}]_\mathcal{M}, [\mathbb{Q}_{\mathrm{Tor}}]_\mathcal{M})$

Complexity results for Reachability in RSMs

# Players	1-box RSMs	1-exit RSMs	multi-exit RSMs
1	NLOGSPACE-complete [12]	PTIME-complete [3]	PTIME-complete [3]
2	PSPACE-complete [20,17]	PTIME-complete [22,15]	EXPTIME-complete [22]

Table 1. Complexity results for reachability objective for RSMs

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Recursive Timed Automata

A recursive timed automaton $\mathcal{T} = (\mathcal{C}, (\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_k))$ is a pair made of a set of clocks $\mathcal{C}$ and a collection of components $(\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_k)$. Each component $\mathcal{T}_i = (N_i, En_i, Ex_i, B_i, Y_i, A_i, X_i, P_i, \mathrm{In}\, v_i, E_i, \rho_i)$ consists of:

- $N_i, En_i, Ex_i, B_i, Y_i, A_i, X_i$ as in RSM
- pass-by-value mapping $P_i : B_i \to 2^C$ that assigns every box the set of clocks that that are passed by value to the component mapped to the box; (The rest of the clocks are assumed to be passed by reference.)
- the invariant condition $Inv_i : Q_i \to \mathcal{Z}$
- the action enabledness function $E_i : Q_i \times A_i \to \mathcal{Z}$; and
- the clock reset function $\rho_i : A_i \to 2^{\mathcal{C}}$.

Example

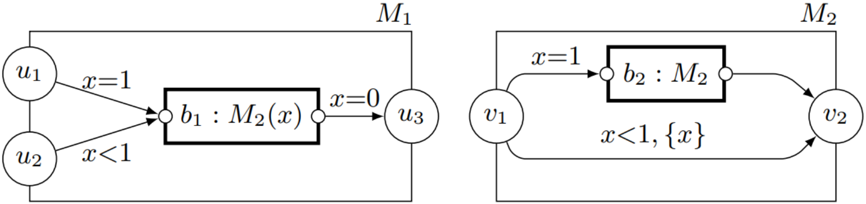

Fig. 2. Example recursive timed automaton

$$\mathcal{T}_i = (N_i, En_i, Ex_i, B_i, Y_i, A_i, X_i, P_i, \mathrm{In}\, v_i, E_i, \rho_i)$$

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Let $\mathcal{T} = (\mathcal{C}, (\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_k))$ be an RTA where each component is of the form $\mathcal{T}_i = (N_i, En_i, Ex_i, B_i, Y_i, A_i, X_i, P_i, \mathrm{Inv}_i, E_i, \rho_i)$. The semantics of $\mathcal{T}$ is a labelled transition system $[\![\mathcal{T}]\!] = (S_\mathcal{T}, A_\mathcal{T}, X_\mathcal{T})$ where:

- $S_\mathcal{T} \subseteq (B \times V)^* \times Q \times V$, the set of states, is such that $(\langle\kappa\rangle, q, \nu) \in S_\mathcal{T}$ if $\nu \in \mathrm{In}\, v(q)$
- $A_\mathcal{T} = \mathbb{R}_\oplus \times A$ is the set of timed actions;
- $X_\mathcal{T} : S_\mathcal{T} \times A_\mathcal{T} \to S_\mathcal{T}$ is the transition function such that for $(\langle\kappa\rangle, q, \nu) \in S_\mathcal{T}$ and $(t, a) \in A_\mathcal{T}$, we have $(\langle\kappa'\rangle, q', \nu') = X_\mathcal{T}((\langle\kappa\rangle, q, \nu), (t, a))$ if and only if the following condition holds:
  - if the vertex q is a call port, i.e. $q = (b, en) \in \mathrm{Call}$ then $t = 0$, the context $\langle\kappa'\rangle = \langle\kappa, (b, \nu)\rangle$, $q' = en$, and $\nu' = \nu$
  - if the vertex q is an exit node, i.e. $q = ex \in Ex$, $\langle\kappa\rangle = \langle\kappa'', (b, \nu'')\rangle$ and let $(b, ex) \in \mathrm{Ret}(b)$, then $t = 0$; $\langle\kappa'\rangle = \langle\kappa''\rangle$; $q' = (b, ex)$; and $\nu' = \nu[P(b) := \nu'']$
  - if vertex q is any other kind of vertex, then $\nu + t' \in \mathrm{Inv}(q)$ for all $t' \in [0, t]$; $\nu + t \in E(q, a)$; and $\langle\kappa'\rangle = \langle\kappa\rangle$, $q' \in X(q, a)$, and $\nu' = (\nu + t)[\rho(a) := \mathbf{0}]$

Undecidability of RTA Termination with ≥ 3 clocks

Theorem

Termination problem is undecidable for recursive timed automata with at least three clocks. Moreover, termination game problem is undecidable for recursive timed automata with at least two clocks.

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Two counter Minsky machines

A Minsky machine $\mathcal{A}$ is a tuple (L, C, D) where: $L = \{\ell_0, \ell_1, \ldots, \ell_n\}$ is the set of states including the distinguished terminal state ℓ_n ; $C = \{c_1, c_2\}$ is the set of two counters; $D = \{\delta_0, \delta_1, \ldots, \delta_{n-1}\}$ is the set of transitions of the following type:

- (increment c)$\delta_i : c := c + 1;$ goto ℓ_k
- (test-and-decrement c)δ_i : if $(c > 0)$ then $(c := c - 1;$ goto ℓ_k else goto ℓ_m

where $c \in C, \delta_i \in D$ and $\ell_k, \ell_m \in L$.
Proof idea: A configuration (ℓ_i, c, d) of a Minsky machine corresponds to the configuration $(\langle \varepsilon \rangle, \ell_i, \nu)$ such that $\nu(x) = 2^{-c} \cdot 3^{-d}$ and $\nu(y) = 0$.

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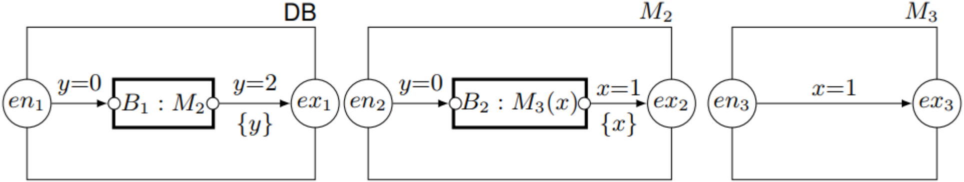
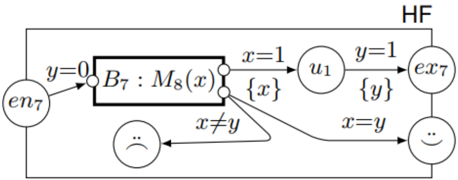
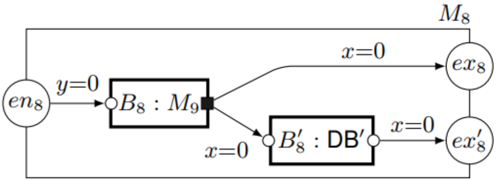


$$
\begin{aligned}
& ((\langle\kappa\rangle, (b, en_1), (x_0, 0)) \rightsquigarrow ((\langle\kappa, b\rangle, en_1, (x_0, 0)) \\
\xrightarrow{y=0}_{0}\; & ((\langle\kappa, b\rangle, (B_1, en_2), (x_0, 0)) \rightsquigarrow ((\langle\kappa, b, B_1\rangle, en_2, (x_0, 0)) \\
\xrightarrow{y=0}_{0}\; & ((\langle\kappa, b, B_1\rangle, (B_2, en_3), (x_0, 0)) \rightsquigarrow ((\langle\kappa, b, B_1, B_2(x_0, 0)\rangle, en_3, (x_0, 0)) \\
\xrightarrow{x=1}_{(1-x_0)}\; & ((\langle\kappa, b, B_1, B_2(x_0, 0)\rangle, ex_3, (1, 1 - x_0)) \rightsquigarrow ((\langle\kappa, b, B_1\rangle, (B_2, ex_3), (x_0, 1 - x_0)) \\
\xrightarrow{x=1,\{x\}}_{(1-x_0)}\; & ((\langle\kappa, b, B_1\rangle, ex_2, (0, 2 - 2 \cdot x_0)) \rightsquigarrow ((\langle\kappa, b\rangle, (B_1, ex_2), (0, 2 - 2 \cdot x_0)) \\
\xrightarrow{y=2,\{y\}}_{(2\cdot x_0)}\; & ((\langle\kappa, b\rangle, ex_1, (2 \cdot x_0, 0)).
\end{aligned}
$$

HF



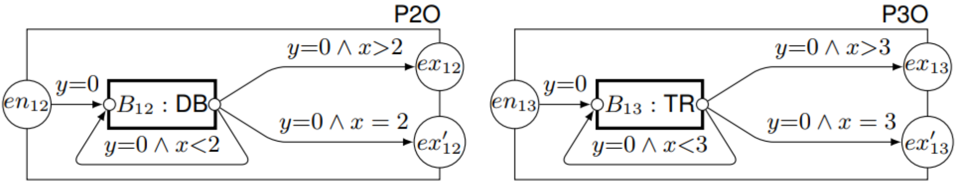


Proof. The main observation here is that, in component HF, starting from the configuration $(\langle\kappa\rangle, (b, en_7), (x_0, 0))$ Achilles has a strategy to terminate only if he chooses to delay the time by $\frac{x_0}{2}$ in component M_9 (called via box B_8). The evolution of the run from $(\langle\kappa\rangle, (b, en_7), (x_0, 0))$ to $(\langle\kappa, b, B_7(x_0, 0), B_8\rangle, en_9, (x_0, 0))$ is straightforward. Now, in component M_9 Achilles can wait for an arbitrary amount of time before taking a transition to ex_9 and resetting clock x. Let us assume that he waits for t time units, and hence $(\langle\kappa, b, B_7(x_0, 0)\rangle, (B_8, ex_9), (0, t))$ is reached which is controlled by Tortoise. Now Tortoise has a choice between making a transition to ex_8 (believing that $t = \frac{x_0}{2}$) or invoking the component B_8' (when suspecting that $t \neq \frac{x_0}{2}$).

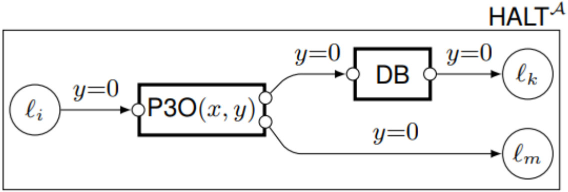
If Tortoise believes that $t = \frac{x_0}{2}$ then he makes a transition to ex_8 and thus the system reaches the configuration $(\langle\kappa, b\rangle, (B_7, ex_8), (x_0, t))$ giving rise to the following run:

$$(\langle\kappa, b\rangle, (B_7, ex_8), (x_0, t)) \xrightarrow{x=1,\{x\}}_{(1-x_0)} (\langle\kappa, b\rangle, u_1, (0, 1 - x_0 + t))$$
$$\xrightarrow{y=1,\{y\}}_{(x_0-t)} (\langle\kappa, b\rangle, ex_7, (x_0 - t, 0)) \rightsquigarrow (\langle\kappa\rangle, (b, ex_7), (x_0 - t, 0)).$$

P2O, P3O


P2O
en_{12}
$y{=}0$
B_{12} : DB
$y{=}0 \wedge x{<}2$
$y{=}0 \wedge x{>}2$
ex_{12}
$y{=}0 \wedge x = 2$
ex'_{12}
P3O
en_{13}
$y{=}0$
B_{13} : TR
$y{=}0 \wedge x{<}3$
$y{=}0 \wedge x{>}3$
ex_{13}
$y{=}0 \wedge x = 3$
ex'_{13}

HALT



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Glitch-free RTA

We say that a recursive timed automaton is glitch-free if for every box either all clocks are passed by value or none is passed by value, i.e. for each $b \in B$ we have that either $P(b) = \mathcal{C}$ or $P(b) = \emptyset$.
Any general recursive timed automaton with one clock is trivially glitch-free.

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Region Equivalence

For every $\mathrm{RTA}\mathcal{T}$ we define regional equivalence relation $\mathcal{E}_R \subseteq S_\mathcal{T} \times S_\mathcal{T}$. For configurations $s = (\langle\kappa\rangle, q, \nu)$ and $s' = (\langle\kappa'\rangle, q', \nu')$, $[s] = [s']$ if:

- $q = q', [\nu] = [\nu']$, and
- $\kappa = (b_1, \nu_1), (b_2, \nu_2), \ldots, (b_n, \nu_n)$ and $\kappa' = (b'_1, \nu'_1), (b'_2, \nu'_2), \ldots, (b'_n, \nu'_n)$ are such that for every $1 \leq i \leq n$ we have $[\nu_i] = [\nu'_i]$ and $b_i = b'_i$.

Bisimulation of Region Equivalence

Given: $s = (\langle\kappa\rangle, q, \nu)$ and $s' = (\langle\kappa'\rangle, q', \nu')$ such that $[s] = [s']$, timed action $(t, a) \in X_T$ such that $X_T(s, (t, a)) = s_a \, (= (\kappa_a, (q_a, \nu_a)))$
Find: (t', a) such that $X_T(s', (t', a)) = s'_a \, (= (\kappa'_a, (q'_a, \nu'_a)))$ and $[s_a] = [s'_a]$

- $q = (b, en) \in Call$: $t = 0$, $\langle\kappa_a\rangle = \langle\kappa, (b, \nu)\rangle, q_a = en$, and $\nu_a = \nu$. $q' = q$ is also a call port. So, $t' = 0$, and $\langle\kappa'_a\rangle = \langle\kappa', (b, \nu')\rangle, q'_a = en$, and $\nu'_a = \nu_a$. Trivially, $[s_a] = [s'_a]$.
- $q = ex \in Ex$: Let $\langle\kappa\rangle = \langle\kappa_*, (b, \nu_*)\rangle$. So, $t = 0$; context $\langle\kappa_a\rangle = \langle\kappa_*\rangle; q_a = (b, ex)$; and $\nu_a = \nu[P(b) := \nu_*]$. Let the context $\langle\kappa'\rangle$ be $\langle\kappa'_*, (b, \nu'_*)\rangle$. $q' = q$ is an exit node. So, $t' = 0, \langle\kappa'_a\rangle = \langle\kappa'_*\rangle$ and $\nu'_a = \nu'[P(b) := \nu'_*]$.
  - $P(b) = \mathcal{C}$: In this case $\nu_a = \nu_*$ and $\nu'_a = \nu'_*$, and since $[\nu_*] = [\nu'_*]$ we get that $[\nu_a] = [\nu'_a]$.
  - $P(b) = \emptyset$. In this case $\nu_a = \nu$ and $\nu'_a = \nu'$, and since $[\nu] = [\nu']$ we get that $[\nu_a] = [\nu'_a]$.
- For other q: Follows by classical region equivalence relation.

Decidability of Reachability on Glitch-free RTA

Theorem

Reachability (termination) problems and games on glitch-free RTA $\mathcal{T}$ can be reduced to solving reachability (termination) problems and games, respectively, on the corresponding region abstraction $\mathcal{T}^{\mathrm{RG}}$.

Complexity results for Reachability in Glitch-free RTAs

# Players	RTAs with 1 clock	RTAs with at least 2 clocks
1	PTIME-complete	EXPTIME-complete
2	EXPTIME-complete	2EXPTIME

Table 2. Complexity results for glitch-free RTAs

# Players	1-box RTAs with 1 global clock	1-box RTAs with at least 2 global clocks
1	PTIME-complete	PSPACE (PSPACE-complete for 3+ clocks)
2	PSPACE-complete	EXPSPACE (and EXPTIME-hard)

Table 3. Complexity results for 1-box RTAs with only global clocks

# Players	1-exit RTAs with 1 local clock	1-exit RTAs with at least 2 local clocks
1	PTIME-complete	EXPTIME-complete
2	PTIME-complete	EXPTIME-complete

Table 4. Complexity results for 1-exit RTAs with only local clocks